
Exercises Prove the following statements:

1. Suppose x and y are natural numbers. Then xy is odd implies that x and y are both odd.
2. If $a, b \in \mathbb{Z}$, then $a^2 - 4b \neq 3$.
3. If a, b, c are integers and $a^2 + b^2 = c^2$, then a or b is even.
4. Suppose x and y are rational numbers such that $x < y$. Prove that there exists $z \in \mathbb{Q}$ such that $x < z < y$.
5. Prove that the following statements are equivalent.

$$A \subseteq B \qquad\qquad A \cap B^c = \emptyset$$

6. The product of any three consecutive natural numbers is divisible by 6.
7. The *Triangle Inequality* for real numbers: $|a + b| \leq |a| + |b|$.
8. For all real numbers a, b: $||a| - |b|| \leq |a - b|$.
9. $\displaystyle\sum_{x=1}^{n} \frac{1}{\sqrt{x}} \leq 2\sqrt{n}$.
10. $(2^{2n-1} + 1)$ is divisible by 3 $\forall n \in \mathbb{N}$.
11. The sum of cubes of three consecutive natural numbers is divisible by 9.
12. $\displaystyle\sum_{k=1}^{n} \frac{1}{k(k+1)} = 1 - \frac{1}{n+1}$.
13. $\displaystyle\sum_{k=1}^{n} \frac{1}{k^2} < 2 - \frac{1}{n}$ for all integers $n \geq 2$.